Pattern Recognition 2024
Assignment#2

April 3, 2024

The format of your report is up to you. In general, your report should clearly show how you have obtained the results and a detailed analysis of your solutions. If you feel a bit inexperienced with writing scientific reports, have a look at the line[1]. I recommend chapter 4 of this document if (like me) English is not your mother language.

Q1. A coin is tossed 100 times and lands heads 62 times. Calculate the maximum likelihood estimation for the probability θ of heads.

Q2. Consider the Bayesian belief network as shown in Figure 1.

1. Which of the following are asserted by the network structure and why?
   (a) $P(B, I, G) = P(B)P(I)P(G)$.
   (b) $P(I|B, L) = P(I|B, L, G)$.
2. Calculate the value of $P(B = f, I = t, L = t, G = f, J = f)$.
3. Calculate the probability that someone will go to jail given that he/she has not broken the law, has been indicted, and his/her lawyer is experienced.

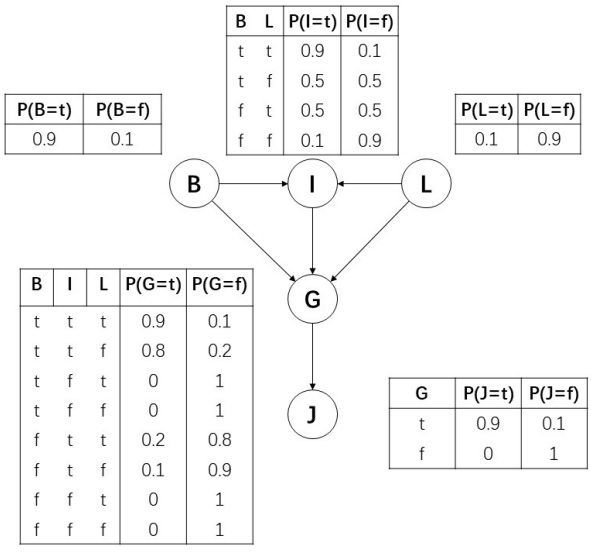

Figure 1: A Bayesian belief network with Bernoulli-distributed variables B = Broke Law, I = Indicted, L = Inexperienced Lawyer, G = Found Guilty, J = Jailed.

[1] http://www.cs.joensuu.fi/pages/whamalai/sciwri/sciwri.pdf

Q3. Define the hidden Markov model $\theta = (A, B, \pi)$ with the three possible states $\{s_1, s_2, s_3\}$, possible observations $\{o_1, o_2, o_3\}$, and the following parameters:

$$\pi = (1, 0, 0)^\top, A = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.5 & 0.0 & 0.5 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}.$$

Let the observed sequences be either $i)(y_1, y_2, y_3) = (o_1, o_2, o_3)$ or $ii)(y_1, y_2, y_3) = (o_1, o_2, o_1)$.

1. What are all possible underlying state sequences (x_1, x_2, x_3) of these two observed sequences?

2. Moreover, calculate $P((y_1, y_2, y_3))$ for both observed sequences.