Pattern Recognition 2024
Assignment#1

March 13, 2024

The format of your report is up to you. In general, your report should clearly show how you have obtained the results and a detailed analysis of your solutions. If you feel a bit inexperienced with writing scientific reports, have a look at the line[1]. I recommend chapter 4 of this document if (like me) English is not your mother language.

Q1. In a two-class three-dimensional classification problem, the feature vectors of each class obey the Gaussian distribution, and the covariance matrix is

$$\boldsymbol{\Sigma}_1 = \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \boldsymbol{\Sigma}_2 = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1/10 \end{pmatrix}$$

The mean vectors $\boldsymbol{\mu}$ for each class are $\boldsymbol{\mu}_1 = [2, 4, 5]^\top$ and $\boldsymbol{\mu}_2 = [1, 2, -4]^\top$, respectively. We assume that the prior probabilities $p(\omega_i)$ of the two classes are equal. Please give the formulation of the decision boundary.
Hints: The discriminant function is given by

$$g_i(\mathbf{x}) = \mathbf{x}^\top \mathbf{W}_i \mathbf{x} + \mathbf{w}_i^\top \mathbf{x} + w_{i0}$$

where

$$\mathbf{W}_i = -\frac{1}{2}\boldsymbol{\Sigma}_i^{-1}$$

$$\mathbf{w}_i = \boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i$$

and

$$w_{i0} = -\frac{1}{2}\boldsymbol{\mu}_i^\top \boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i - \frac{1}{2}\ln|\boldsymbol{\Sigma}_i| + \ln p(\omega_i)$$

[1] http://www.cs.joensuu.fi/pages/whamalai/sciwri/sciwri.pdf

Q2. Imagine you are a manager at a company that produces a certain component for electronic devices. Your company has two suppliers, Supplier A and Supplier B, with preference of 0.6 and 0.4, respectively. For products manufactured by Supplier A, the probability of using high-quality raw materials is 0.9, while for Supplier B, the probability is 0.7. You randomly select a product and it is found to be made of low-quality raw materials. Based on this information, try to solve the following question.

(a) Determine from which supplier you are more likely to have purchased the material, and provide your reasoning.

(b) The loss function is defined as: λ_{11}=10, λ_{12}=1, λ_{21}=9, λ_{22}=3. Considering the minimum risk Bayesian decision, determine from which supplier you are more likely to have purchased the material?