# Appendix for Learning Label-Specific Multiple Local Metrics for Multi-Label Classification

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## A The complete procedure of LSMM

The complete procedures of LSMM-SE and LSMM-CL are summarized in Algorithm 1 and 2 respectively.

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**Algorithm 1** The pseudo-code of LSMM-SE

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**Input**: Training set $\mathcal{D} = \{(\boldsymbol{x}_i, Y_i) \mid 1 \leq i \leq n\}(\mathcal{X} = \mathbb{R}^d, \mathcal{Y} = \{l_1, l_2, \ldots, l_q\}, \boldsymbol{x}_i \in \mathcal{X}, Y_i \subseteq \mathcal{Y})$, regularization parameters $\lambda_1, \lambda_2$, threshold parameters $\alpha, \gamma$, number of targets and imposters $k_t, k_i$, and maximum number of iterations $I$.

**Output**: The learned global metric $\mathbf{L}_0$ and label-specific multiple local metrics $\left\{\mathbf{L}_p^1, \mathbf{L}_p^0\right\}_{p=1}^q$.

Initialize $\mathbf{L}_0$ randomly;
Initialize $\left\{\mathbf{L}_p^1, \mathbf{L}_p^0\right\}_{p=1}^q$ with $\mathbf{0}$;
**for** $p = 1$ to $q$ **do**
  Generate positive set $\mathcal{P}_p$ and negative set $\mathcal{N}_p$ according to Eq.(2);
  **for** $i = 1$ to $n$ **do**
    Find $k_t$ nearest targets and $k_i$ nearest imposters of $\boldsymbol{x}_i$ to generate label-specific side information $\mathcal{T}_p$;
  **end for**
**end for**
**repeat**
  Optimize Eq.(4) over $\mathbf{L}_0$ and $\left\{\mathbf{L}_p^1, \mathbf{L}_p^0\right\}_{p=1}^q$ with L-BFGS algorithm according to Eq.(8-10);
**until** convergence or $I$ being reached
Return $\mathbf{L}_0$ and $\left\{\mathbf{L}_p^1, \mathbf{L}_p^0\right\}_{p=1}^q$

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**Algorithm 2** The pseudo-code of LSMM-CL

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**Input**: Training set $\mathcal{D} = \{(\boldsymbol{x}_i, Y_i) \mid 1 \leq i \leq n\}(\mathcal{X} = \mathbb{R}^d, \mathcal{Y} = \{l_1, l_2, \ldots, l_q\}, \boldsymbol{x}_i \in \mathcal{X}, Y_i \subseteq \mathcal{Y})$, regularization parameters $\lambda_1, \lambda_2$, threshold parameters $\alpha, \gamma$, number of targets and imposters $k_t, k_i$, number of cluster $C$, and maximum number of iterations $I$.

**Output**: The learned global metric $\mathbf{L}_0$ and label-specific multiple local metrics $\left\{\mathbf{L}_p^1, \mathbf{L}_p^2, \ldots, \mathbf{L}_p^C\right\}_{p=1}^q$.

Initialize $\mathbf{L}_0$ randomly;
Initialize $\left\{\mathbf{L}_p^1, \mathbf{L}_p^2, \ldots, \mathbf{L}_p^C\right\}_{p=1}^q$ with $\mathbf{0}$;
Divide $\mathcal{D}$ into $C$ clusters via $k$-means;
**for** $p = 1$ to $q$ **do**
  Generate positive set $\mathcal{P}_p$ and negative set $\mathcal{N}_p$ according to Eq.(2);
  **for** $i = 1$ to $n$ **do**
    Find $k_t$ nearest targets and $k_i$ nearest imposters of $\boldsymbol{x}_i$ to generate label-specific side information $\mathcal{T}_p$;
  **end for**
**end for**
**repeat**
  Optimize Eq.(7) over $\mathbf{L}_0$ and $\left\{\mathbf{L}_p^1, \mathbf{L}_p^2, \ldots, \mathbf{L}_p^C\right\}_{p=1}^q$ with L-BFGS algorithm according to Eq.(13-14);
**until** convergence or $I$ being reached
Return $\mathbf{L}_0$ and $\left\{\mathbf{L}_p^1, \mathbf{L}_p^2, \ldots, \mathbf{L}_p^C\right\}_{p=1}^q$

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## B Experiments

### B.1 More Experimental Results with SOTA Multi-Label Metric Learning Algorithms

Table B.1 reports detailed experimental results with state-of-art multi-label metric learning algorithms in terms of *Coverage*, *Average Precision*, *Macro-F1* and *Macro-average AUC* which are not covered in the main body due to page limit. Furthermore, pairwise $t$-test at 0.05 significance is conducted and the corresponding win/tie/loss counts are reported in Table B.3. These results clearly demonstrate the superiority of our proposed LSMM framework in learning effective similarity metrics for multi-label classification.

### B.2 More Experimental Results with SOTA Non-Metric Learning Multi-Label Classification approahces

Table B.2 reports detailed experimental results with state-of-the-art non-metric learning multi-label classification approaches in terms of *Coverage*, *Average Precision*, *Macro-F1* and *Macro-average AUC*. The corresponding win/tie/loss counts (pairwise $t$-test at 0.05 significant level) is reported in Table B.4. The results once again validate the superiority of our proposed LSMM framework. Furthermore, we observe that when coupled with LSMM, simple BR-KNN and ML-KNN also have the potential to approach or even surpass state-of-the-art multi-label classification methods.

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| Compared Algorithms | Datasets | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | emotions | birds | medical | enron | image | scene | slashdot | arts | education |
| | | | | | *Coverage* ↓ | | | | |
| BR-kNN | 0.378±0.032 | 0.216±0.037 | 0.095±0.025 | 0.334±0.026 | 0.192±0.023 | 0.085±0.012 | 0.310±0.020 | 0.321±0.023 | 0.187±0.010 |
| BR-kNN-LM | 0.364±0.032 | 0.220±0.036 | 0.083±0.026 | **0.310±0.017** | 0.196±0.019 | 0.089±0.013 | 0.259±0.021 | 0.263±0.012 | 0.166±0.009 |
| BR-kNN-LJE | 0.330±0.026 | 0.218±0.038 | 0.094±0.031 | 0.378±0.029 | 0.211±0.020 | 0.108±0.016 | 0.284±0.015 | 0.277±0.012 | 0.170±0.010 |
| BR-kNN-COMMU | 0.378±0.032 | 0.215±0.038 | 0.081±0.026 | 0.334±0.026 | 0.192±0.023 | 0.085±0.012 | 0.305±0.021 | 0.323±0.027 | 0.187±0.010 |
| BR-kNN-LIMIC | 0.312±0.032 | 0.186±0.024 | 0.072±0.018 | 0.339±0.018 | 0.184±0.026 | 0.084±0.007 | 0.274±0.021 | 0.307±0.015 | 0.161±0.014 |
| BR-kNN-LSMM-SE | **0.299±0.026** | **0.175±0.024** | 0.067±0.017 | 0.326±0.018 | **0.176±0.018** | 0.085±0.012 | 0.233±0.018 | 0.279±0.021 | 0.147±0.015 |
| BR-kNN-LSMM-CL | 0.304±0.025 | 0.179±0.022 | **0.065±0.015** | 0.330±0.014 | 0.182±0.017 | **0.081±0.013** | **0.220±0.020** | **0.260±0.014** | **0.145±0.013** |
| ML-kNN | 0.377±0.026 | 0.191±0.029 | 0.077±0.011 | 0.246±0.021 | 0.192±0.021 | 0.077±0.011 | 0.197±0.012 | 0.205±0.008 | 0.124±0.005 |
| ML-kNN-LM | 0.360±0.033 | 0.190±0.033 | 0.049±0.019 | 0.246±0.014 | 0.193±0.018 | 0.079±0.010 | 0.149±0.009 | 0.188±0.007 | 0.106±0.005 |
| ML-kNN-LJE | 0.329±0.030 | 0.192±0.036 | 0.070±0.021 | 0.283±0.020 | 0.213±0.021 | 0.106±0.015 | 0.212±0.012 | 0.205±0.008 | 0.118±0.006 |
| ML-kNN-COMMU | 0.377±0.026 | 0.191±0.029 | 0.075±0.017 | 0.246±0.020 | 0.192±0.021 | 0.077±0.011 | 0.173±0.010 | 0.205±0.008 | 0.114±0.005 |
| ML-kNN-LIMIC | 0.314±0.015 | 0.174±0.026 | **0.046±0.013** | 0.247±0.019 | 0.179±0.023 | 0.073±0.007 | 0.160±0.010 | 0.197±0.006 | 0.108±0.005 |
| ML-kNN-LSMM-SE | **0.297±0.026** | **0.163±0.022** | 0.053±0.015 | 0.245±0.015 | **0.173±0.018** | 0.073±0.006 | 0.127±0.009 | 0.194±0.007 | 0.105±0.002 |
| ML-kNN-LSMM-CL | 0.300±0.030 | 0.167±0.026 | 0.050±0.012 | **0.242±0.014** | 0.176±0.016 | **0.072±0.005** | **0.138±0.011** | **0.182±0.006** | **0.102±0.003** |
| | | | | | *Average Precision* ↑ | | | | |
| BR-kNN | 0.700±0.049 | 0.358±0.043 | 0.801±0.025 | 0.581±0.022 | 0.795±0.021 | 0.854±0.013 | 0.428±0.023 | 0.420±0.021 | 0.577±0.015 |
| BR-kNN-LM | 0.711±0.038 | 0.332±0.052 | 0.843±0.032 | 0.641±0.019 | 0.783±0.019 | 0.856±0.021 | 0.665±0.020 | 0.585±0.013 | 0.602±0.016 |
| BR-kNN-LJE | 0.773±0.041 | 0.341±0.061 | 0.782±0.041 | 0.538±0.034 | 0.769±0.021 | 0.812±0.022 | 0.481±0.022 | 0.536±0.020 | 0.561±0.013 |
| BR-kNN-COMMU | 0.700±0.049 | 0.359±0.044 | 0.796±0.027 | 0.579±0.022 | 0.795±0.021 | 0.854±0.013 | 0.430±0.023 | 0.418±0.022 | 0.577±0.015 |
| BR-kNN-LIMIC | 0.788±0.032 | 0.392±0.056 | 0.854±0.025 | 0.639±0.026 | 0.807±0.020 | 0.860±0.012 | 0.598±0.021 | 0.529±0.019 | 0.598±0.019 |
| BR-kNN-LSMM-SE | 0.795±0.039 | **0.438±0.063** | **0.882±0.036** | 0.637±0.029 | 0.817±0.023 | 0.857±0.012 | 0.678±0.021 | 0.593±0.020 | 0.612±0.015 |
| BR-kNN-LSMM-CL | **0.797±0.038** | 0.427±0.058 | 0.878±0.030 | **0.648±0.032** | **0.820±0.018** | **0.868±0.017** | **0.677±0.025** | **0.602±0.018** | **0.619±0.017** |
| ML-kNN | 0.712±0.042 | 0.414±0.052 | 0.819±0.020 | 0.645±0.020 | 0.792±0.018 | 0.872±0.017 | 0.535±0.018 | 0.532±0.017 | 0.618±0.016 |
| ML-kNN-LM | 0.719±0.019 | 0.405±0.056 | 0.853±0.034 | 0.680±0.020 | 0.790±0.019 | 0.866±0.020 | 0.705±0.018 | 0.611±0.013 | 0.632±0.017 |
| ML-kNN-LJE | 0.767±0.043 | 0.393±0.050 | 0.778±0.041 | 0.571±0.029 | 0.765±0.022 | 0.819±0.024 | 0.508±0.021 | 0.557±0.020 | 0.588±0.008 |
| ML-kNN-COMMU | 0.712±0.042 | 0.414±0.052 | 0.810±0.024 | 0.646±0.021 | 0.792±0.018 | 0.872±0.017 | 0.537±0.022 | 0.529±0.016 | 0.618±0.016 |
| ML-kNN-LIMIC | 0.783±0.023 | 0.443±0.054 | 0.872±0.022 | 0.675±0.020 | 0.812±0.022 | 0.883±0.010 | 0.670±0.018 | 0.581±0.015 | 0.639±0.015 |
| ML-kNN-LSMM-SE | **0.794±0.036** | **0.472±0.049** | **0.903±0.038** | 0.698±0.026 | 0.812±0.019 | 0.886±0.016 | 0.711±0.015 | 0.635±0.013 | 0.640±0.015 |
| ML-kNN-LSMM-CL | **0.794±0.043** | 0.458±0.048 | 0.897±0.035 | **0.705±0.032** | **0.820±0.023** | **0.890±0.015** | **0.718±0.017** | **0.640±0.015** | **0.649±0.017** |
| | | | | | *Macro-F1* ↑ | | | | |
| BR-kNN | 0.463±0.045 | 0.038±0.018 | 0.169±0.018 | 0.092±0.013 | 0.599±0.033 | 0.731±0.016 | 0.067±0.004 | 0.126±0.017 | 0.137±0.015 |
| BR-kNN-LM | 0.490±0.033 | 0.060±0.035 | 0.320±0.051 | 0.166±0.020 | 0.597±0.034 | 0.744±0.035 | **0.403±0.019** | 0.191±0.009 | 0.162±0.017 |
| BR-kNN-LJE | 0.583±0.047 | 0.034±0.016 | 0.273±0.027 | 0.061±0.012 | 0.554±0.040 | 0.664±0.028 | 0.098±0.013 | 0.148±0.014 | 0.122±0.015 |
| BR-kNN-COMMU | 0.463±0.045 | 0.047±0.025 | 0.297±0.040 | 0.109±0.015 | 0.599±0.033 | 0.731±0.016 | 0.072±0.005 | 0.136±0.017 | 0.176±0.023 |
| BR-kNN-LIMIC | 0.609±0.016 | 0.093±0.028 | 0.384±0.031 | 0.142±0.020 | 0.622±0.034 | 0.760±0.024 | 0.342±0.018 | 0.185±0.019 | 0.157±0.016 |
| BR-kNN-LSMM-SE | **0.651±0.037** | **0.107±0.032** | **0.541±0.060** | 0.162±0.015 | 0.631±0.027 | **0.763±0.023** | 0.397±0.017 | 0.201±0.018 | 0.172±0.015 |
| BR-kNN-LSMM-CL | 0.620±0.033 | 0.101±0.029 | 0.526±0.048 | **0.178±0.023** | **0.638±0.039** | 0.757±0.023 | 0.402±0.016 | **0.213±0.019** | **0.179±0.021** |
| ML-kNN | 0.361±0.050 | 0.010±0.007 | 0.210±0.027 | 0.086±0.010 | 0.581±0.028 | 0.748±0.022 | 0.127±0.017 | 0.098±0.010 | 0.126±0.023 |
| ML-kNN-LM | 0.438±0.036 | 0.016±0.011 | 0.294±0.041 | 0.146±0.010 | 0.573±0.031 | 0.746±0.031 | 0.413±0.027 | 0.187±0.022 | 0.161±0.028 |
| ML-kNN-LJE | 0.530±0.052 | 0.009±0.011 | 0.245±0.044 | 0.045±0.012 | 0.530±0.037 | 0.651±0.023 | 0.099±0.015 | 0.125±0.012 | 0.101±0.018 |
| ML-kNN-COMMU | 0.361±0.050 | 0.013±0.008 | 0.364±0.061 | 0.102±0.009 | 0.581±0.028 | 0.748±0.022 | 0.136±0.016 | 0.108±0.013 | 0.161±0.035 |
| ML-kNN-LIMIC | 0.587±0.028 | 0.071±0.030 | 0.384±0.040 | 0.134±0.020 | 0.635±0.031 | 0.763±0.019 | 0.398±0.019 | 0.197±0.018 | 0.182±0.032 |
| ML-kNN-LSMM-SE | **0.604±0.031** | **0.099±0.026** | **0.556±0.052** | 0.157±0.018 | 0.632±0.034 | **0.765±0.025** | 0.408±0.023 | 0.221±0.027 | 0.198±0.026 |
| ML-kNN-LSMM-CL | 0.597±0.027 | 0.097±0.037 | 0.551±0.049 | **0.165±0.019** | **0.649±0.035** | 0.761±0.027 | **0.416±0.025** | **0.226±0.024** | **0.207±0.027** |
| | | | | | *Macro-average AUC* ↑ | | | | |
| BR-kNN | 0.737±0.041 | 0.670±0.040 | 0.880±0.032 | 0.667±0.029 | 0.842±0.020 | 0.936±0.008 | 0.665±0.016 | 0.618±0.020 | 0.696±0.016 |
| BR-kNN-LM | 0.742±0.024 | 0.658±0.056 | 0.879±0.050 | 0.675±0.018 | 0.829±0.019 | 0.925±0.010 | 0.791±0.021 | 0.663±0.016 | 0.660±0.024 |
| BR-kNN-LJE | 0.800±0.034 | 0.646±0.051 | 0.855±0.041 | 0.582±0.017 | 0.817±0.021 | 0.904±0.016 | 0.661±0.021 | 0.643±0.010 | 0.656±0.020 |
| BR-kNN-COMMU | 0.737±0.041 | 0.670±0.040 | 0.866±0.039 | 0.666±0.029 | 0.842±0.020 | 0.936±0.008 | 0.668±0.017 | 0.617±0.020 | 0.697±0.013 |
| BR-kNN-LIMIC | 0.827±0.022 | 0.752±0.031 | 0.894±0.038 | 0.655±0.017 | 0.850±0.022 | 0.929±0.006 | 0.720±0.017 | 0.635±0.016 | 0.683±0.015 |
| BR-kNN-LSMM-SE | **0.837±0.030** | **0.767±0.029** | 0.928±0.041 | 0.682±0.031 | **0.853±0.018** | 0.936±0.007 | 0.859±0.016 | **0.748±0.024** | 0.742±0.033 |
| BR-kNN-LSMM-CL | 0.832±0.030 | 0.762±0.032 | **0.945±0.037** | **0.698±0.022** | **0.860±0.024** | **0.940±0.005** | **0.876±0.024** | 0.745±0.027 | **0.759±0.029** |
| ML-kNN | 0.720±0.041 | 0.666±0.044 | 0.876±0.032 | 0.657±0.024 | 0.835±0.019 | 0.934±0.008 | 0.661±0.016 | 0.613±0.020 | 0.695±0.015 |
| ML-kNN-LM | 0.729±0.025 | 0.642±0.062 | 0.878±0.050 | 0.668±0.017 | 0.829±0.018 | 0.925±0.009 | 0.790±0.021 | 0.660±0.018 | 0.659±0.024 |
| ML-kNN-LJE | 0.788±0.035 | 0.636±0.049 | 0.852±0.041 | 0.570±0.014 | 0.810±0.021 | 0.901±0.016 | 0.657±0.019 | 0.638±0.009 | 0.655±0.020 |
| ML-kNN-COMMU | 0.720±0.041 | 0.666±0.045 | 0.862±0.040 | 0.657±0.023 | 0.835±0.019 | 0.933±0.008 | 0.653±0.013 | 0.612±0.020 | 0.693±0.014 |
| ML-kNN-LIMIC | 0.815±0.022 | 0.743±0.032 | 0.893±0.038 | 0.655±0.016 | 0.845±0.021 | 0.928±0.006 | 0.720±0.017 | 0.633±0.017 | 0.654±0.016 |
| ML-kNN-LSMM-SE | **0.828±0.028** | **0.763±0.022** | 0.927±0.043 | 0.685±0.025 | 0.848±0.021 | 0.934±0.006 | 0.862±0.023 | 0.745±0.031 | 0.753±0.027 |
| ML-kNN-LSMM-CL | 0.824±0.029 | 0.758±0.019 | 0.938±0.037 | **0.699±0.023** | **0.858±0.021** | **0.941±0.007** | **0.879±0.025** | **0.752±0.026** | **0.768±0.031** |

Table B.1: Predictive performance (mean±std) of $\mathcal{A}$ ($\mathcal{A} \in \{$BR-kNN, ML-kNN$\}$) coupled with our proposed approaches and state-of-the-art multi-label metric learning approaches in terms of *Coverage*, *Average Precision*, *Macro-F1*, and *Macro-average AUC*. ↑ (↓) indicates the larger (smaller) the value, the better the performance. The best and second best results are highlighted in **boldface** and underline respectively.

| Compared Algorithms | Datasets | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | emotions | birds | medical | enron | image | scene | slashdot | arts | education |
| | | | | | *Coverage* ↓ | | | | |
| Lift | 0.368±0.035 | 0.204±0.032 | 0.052±0.015 | **0.237±0.013** | 0.181±0.022 | 0.075±0.010 | 0.130±0.007 | **0.173±0.006** | 0.112±0.005 |
| Reliab | 0.393±0.023 | 0.217±0.031 | 0.063±0.013 | 0.276±0.030 | 0.199±0.027 | 0.091±0.009 | 0.137±0.019 | 0.184±0.011 | 0.130±0.009 |
| Wrap | 0.372±0.030 | 0.189±0.048 | 0.059±0.009 | 0.259±0.021 | 0.192±0.013 | 0.086±0.009 | 0.149±0.012 | 0.184±0.015 | 0.128±0.008 |
| Homi | 0.385±0.025 | 0.204±0.034 | 0.055±0.016 | 0.260±0.025 | 0.188±0.020 | 0.085±0.008 | 0.132±0.013 | 0.234±0.013 | 0.140±0.004 |
| Br-knn-Lsmm-se | 0.299±0.026 | 0.175±0.024 | 0.067±0.017 | 0.326±0.018 | 0.176±0.018 | 0.085±0.012 | 0.233±0.018 | 0.279±0.021 | 0.147±0.015 |
| Br-knn-Lsmm-cl | 0.304±0.025 | 0.179±0.022 | 0.065±0.015 | 0.330±0.014 | 0.182±0.017 | 0.081±0.013 | 0.220±0.020 | 0.260±0.014 | 0.145±0.013 |
| Ml-knn-Lsmm-se | **0.297±0.026** | **0.163±0.022** | 0.053±0.015 | 0.245±0.015 | **0.173±0.018** | 0.073±0.006 | **0.127±0.009** | 0.194±0.007 | 0.105±0.002 |
| Ml-knn-Lsmm-cl | 0.300±0.030 | 0.167±0.026 | **0.050±0.012** | 0.242±0.014 | 0.176±0.016 | **0.072±0.005** | 0.138±0.011 | 0.182±0.006 | **0.102±0.003** |
| | | | | | *Average Precision* ↑ | | | | |
| Lift | 0.718±0.046 | 0.363±0.046 | 0.856±0.028 | **0.713±0.018** | 0.817±0.020 | 0.882±0.014 | 0.687±0.019 | 0.626±0.014 | 0.633±0.011 |
| Reliab | 0.676±0.041 | 0.362±0.011 | 0.901±0.024 | 0.643±0.024 | 0.784±0.021 | 0.860±0.013 | 0.713±0.026 | 0.605±0.012 | 0.637±0.014 |
| Wrap | 0.714±0.032 | 0.416±0.035 | 0.902±0.014 | 0.690±0.022 | 0.788±0.026 | 0.872±0.014 | **0.725±0.023** | 0.636±0.017 | 0.646±0.016 |
| Homi | 0.698±0.041 | 0.398±0.032 | 0.890±0.025 | 0.660±0.019 | 0.795±0.017 | 0.866±0.013 | 0.685±0.022 | 0.585±0.015 | 0.641±0.011 |
| Br-knn-Lsmm-se | 0.795±0.039 | 0.438±0.063 | 0.882±0.036 | 0.637±0.029 | 0.817±0.023 | 0.857±0.012 | 0.678±0.021 | 0.593±0.020 | 0.612±0.015 |
| Br-knn-Lsmm-cl | **0.797±0.038** | 0.427±0.058 | 0.878±0.030 | 0.648±0.032 | **0.820±0.018** | 0.868±0.017 | 0.677±0.025 | 0.602±0.018 | 0.619±0.017 |
| Ml-knn-Lsmm-se | 0.794±0.036 | **0.472±0.049** | **0.903±0.038** | 0.698±0.026 | 0.812±0.019 | 0.886±0.016 | 0.711±0.015 | 0.635±0.013 | 0.640±0.015 |
| Ml-knn-Lsmm-cl | 0.794±0.043 | 0.458±0.048 | 0.897±0.035 | 0.705±0.032 | **0.820±0.023** | **0.890±0.015** | 0.718±0.017 | **0.640±0.015** | **0.649±0.017** |
| | | | | | *Macro-F1* ↑ | | | | |
| Lift | 0.432±0.040 | 0.024±0.018 | 0.310±0.061 | 0.150±0.015 | 0.606±0.036 | 0.759±0.016 | 0.250±0.015 | 0.123±0.009 | 0.156±0.011 |
| Reliab | 0.538±0.047 | 0.098±0.012 | 0.473±0.038 | 0.132±0.008 | 0.555±0.016 | 0.687±0.015 | 0.396±0.016 | 0.151±0.003 | 0.176±0.004 |
| Wrap | 0.318±0.036 | 0.093±0.012 | 0.529±0.022 | 0.148±0.006 | 0.501±0.036 | **0.773±0.029** | 0.373±0.013 | 0.192±0.012 | 0.194±0.005 |
| Homi | 0.488±0.028 | 0.068±0.023 | 0.511±0.061 | 0.126±0.006 | **0.650±0.034** | 0.759±0.025 | 0.378±0.026 | 0.186±0.020 | 0.183±0.016 |
| Br-knn-Lsmm-se | **0.651±0.037** | **0.107±0.032** | 0.541±0.060 | 0.162±0.015 | 0.631±0.027 | 0.763±0.023 | 0.397±0.017 | 0.201±0.018 | 0.172±0.015 |
| Br-knn-Lsmm-cl | 0.620±0.033 | 0.101±0.029 | 0.526±0.048 | **0.178±0.023** | 0.638±0.039 | 0.757±0.023 | 0.402±0.016 | 0.213±0.019 | 0.179±0.021 |
| Ml-knn-Lsmm-se | 0.604±0.031 | 0.099±0.026 | **0.556±0.052** | 0.157±0.018 | 0.632±0.034 | 0.765±0.025 | 0.408±0.023 | 0.221±0.027 | 0.198±0.026 |
| Ml-knn-Lsmm-cl | 0.597±0.027 | 0.097±0.037 | 0.551±0.049 | 0.165±0.019 | 0.649±0.035 | 0.761±0.027 | **0.416±0.025** | **0.226±0.024** | **0.207±0.027** |
| | | | | | *Macro-average AUC* ↑ | | | | |
| Lift | 0.740±0.033 | 0.713±0.044 | 0.914±0.035 | 0.637±0.016 | 0.856±0.022 | 0.930±0.007 | 0.841±0.018 | 0.738±0.033 | 0.760±0.022 |
| Reliab | 0.718±0.034 | 0.685±0.021 | 0.939±0.026 | 0.667±0.033 | 0.832±0.024 | 0.927±0.006 | 0.861±0.017 | 0.743±0.029 | 0.735±0.032 |
| Wrap | 0.715±0.032 | 0.722±0.047 | 0.913±0.061 | 0.678±0.026 | 0.830±0.022 | 0.918±0.009 | 0.807±0.041 | 0.717±0.017 | 0.711±0.040 |
| Homi | 0.709±0.026 | 0.720±0.041 | 0.904±0.044 | 0.669±0.024 | 0.842±0.027 | 0.925±0.005 | 0.832±0.018 | 0.731±0.036 | 0.741±0.017 |
| Br-knn-Lsmm-se | **0.837±0.030** | **0.767±0.029** | 0.928±0.041 | 0.682±0.031 | 0.853±0.018 | 0.936±0.007 | 0.859±0.016 | 0.748±0.024 | 0.742±0.033 |
| Br-knn-Lsmm-cl | 0.832±0.030 | 0.762±0.032 | **0.945±0.037** | 0.698±0.022 | **0.860±0.024** | 0.940±0.005 | 0.876±0.024 | 0.745±0.027 | 0.759±0.029 |
| Ml-knn-Lsmm-se | 0.828±0.028 | 0.763±0.022 | 0.927±0.043 | 0.685±0.025 | 0.848±0.021 | 0.934±0.006 | 0.862±0.023 | 0.745±0.031 | 0.753±0.027 |
| Ml-knn-Lsmm-cl | 0.824±0.029 | 0.758±0.019 | 0.938±0.037 | **0.699±0.023** | 0.858±0.021 | **0.941±0.007** | **0.879±0.025** | **0.752±0.026** | **0.768±0.031** |

Table B.2: Predictive performance (mean±std) of $\mathcal{A}$ ($\mathcal{A} \in \{$Br-knn, Ml-knn$\}$) coupled with our proposed approaches and state-of-the-art non-metric learning multi-label classification approaches in terms of *Coverage*, *Average Precision*, *Macro-F1*, and *Macro-average AUC*. ↑ (↓) indicates the larger (smaller) the value, the better the performance. The best and second best results are highlighted in **boldface** and underline respectively.

| Metrics | $\mathcal{A}$-LSMM-SE ($\mathcal{A}$ = BR-KNN) **against** | | | | | $\mathcal{A}$-LSMM-SE ($\mathcal{A}$ = ML-KNN) **against** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{A}$-LM | $\mathcal{A}$-LJE | $\mathcal{A}$-COMMU | $\mathcal{A}$-LIMIC | $\mathcal{A}$ | $\mathcal{A}$-LM | $\mathcal{A}$-LJE | $\mathcal{A}$-COMMU | $\mathcal{A}$-LIMIC |
| *Hamming Loss* | 7/2/0 | 5/3/1 | 9/0/0 | 7/2/0 | 5/4/0 | 8/1/0 | 6/3/0 | 9/0/0 | 8/1/0 | 6/3/0 |
| *Ranking Loss* | 8/1/0 | 7/1/1 | 8/1/0 | 8/1/0 | 6/2/1 | 8/1/0 | 7/2/0 | 9/0/0 | 8/1/0 | 6/2/1 |
| *Coverage* | 8/1/0 | 7/0/2 | 8/1/0 | 8/1/0 | 8/1/0 | 9/0/0 | 5/2/2 | 9/0/0 | 8/1/0 | 5/3/1 |
| *Average precision* | 8/1/0 | 7/1/1 | 9/0/0 | 8/1/0 | 7/2/0 | 9/0/0 | 9/0/0 | 9/0/0 | 9/0/0 | 6/3/0 |
| *Macro-F1* | 9/0/0 | 7/0/2 | 9/0/0 | 8/0/1 | 8/1/0 | 9/0/0 | 8/0/1 | 9/0/0 | 9/0/0 | 7/2/0 |
| *Macro-averaging AUC* | 8/1/0 | 8/1/0 | 8/1/0 | 8/1/0 | 8/1/0 | 8/1/0 | 9/0/0 | 9/0/0 | 9/0/0 | 9/0/0 |
| **In Total** | **48/6/0** | **41/6/7** | **51/3/0** | **47/6/1** | **42/11/1** | **51/3/0** | **44/7/3** | **54/0/0** | **51/3/0** | **39/13/2** |

| Metrics | $\mathcal{A}$-LSMM-CL ($\mathcal{A}$ = BR-KNN) **against** | | | | | $\mathcal{A}$-LSMM-CL ($\mathcal{A}$ = ML-KNN) **against** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{A}$-LM | $\mathcal{A}$-LJE | $\mathcal{A}$-COMMU | $\mathcal{A}$-LIMIC | $\mathcal{A}$ | $\mathcal{A}$-LM | $\mathcal{A}$-LJE | $\mathcal{A}$-COMMU | $\mathcal{A}$-LIMIC |
| *Hamming Loss* | 8/1/0 | 7/2/0 | 9/0/0 | 8/1/0 | 7/2/0 | 8/1/0 | 7/2/0 | 9/0/0 | 8/1/0 | 6/3/0 |
| *Ranking Loss* | 9/0/0 | 9/0/0 | 9/0/0 | 8/1/0 | 9/0/0 | 8/0/1 | 7/1/1 | 9/0/0 | 8/1/0 | 6/2/1 |
| *Coverage* | 7/2/0 | 6/2/1 | 9/0/0 | 7/2/0 | 7/2/0 | 8/1/0 | 6/3/0 | 9/0/0 | 8/1/0 | 5/3/1 |
| *Average precision* | 9/0/0 | 8/1/0 | 9/0/0 | 9/0/0 | 8/1/0 | 9/0/0 | 9/0/0 | 9/0/0 | 9/0/0 | 8/1/0 |
| *Macro-F1* | 9/0/0 | 8/1/0 | 9/0/0 | 8/1/0 | 8/1/0 | 9/0/0 | 8/1/0 | 9/0/0 | 9/0/0 | 7/1/1 |
| *Macro-averaging AUC* | 8/1/0 | 9/0/0 | 9/0/0 | 8/1/0 | 7/2/0 | 8/1/0 | 9/0/0 | 9/0/0 | 9/0/0 | 9/0/0 |
| **In Total** | **50/4/0** | **47/6/1** | **54/0/0** | **48/6/0** | **46/8/0** | **50/3/1** | **46/7/1** | **54/0/0** | **51/3/0** | **41/10/3** |

Table B.3: Win/tie/loss counts (pairwise $t$-test at 0.05 significant level) for LSMM-SE and LSMM-CL against other compared state-of-the-art multi-label metric learning algorithms coupled with $\mathcal{A}$ ($\mathcal{A} \in \{$BR-KNN, ML-KNN$\}$).

| Metrics | $\mathcal{A}$-LSMM-SE ($\mathcal{A}$ = BR-KNN) **against** | | | | $\mathcal{A}$-LSMM-SE ($\mathcal{A}$ = ML-KNN) **against** | | | |
|---|---|---|---|---|---|---|---|---|
| | LIFT | RELIAB | WRAP | HOMI | LIFT | RELIAB | WRAP | HOMI |
| *Hamming Loss* | 3/6/0 | 6/3/0 | 4/4/1 | 4/5/0 | 3/6/0 | 7/2/0 | 4/3/2 | 4/5/0 |
| *Ranking Loss* | 1/1/7 | 2/1/6 | 2/0/7 | 2/0/7 | 4/4/1 | 8/0/1 | 7/0/2 | 9/0/0 |
| *Coverage* | 3/0/6 | 3/2/4 | 3/1/5 | 3/1/5 | 4/3/2 | 8/0/1 | 8/0/1 | 8/1/0 |
| *Average precision* | 3/2/4 | 3/3/3 | 3/0/6 | 3/4/2 | 6/2/1 | 5/4/0 | 4/2/3 | 8/1/0 |
| *Macro-F1* | 8/1/0 | 6/1/2 | 7/0/2 | 6/1/2 | 9/0/0 | 8/1/0 | 6/2/1 | 7/1/1 |
| *Macro-averaging AUC* | 7/1/1 | 5/3/1 | 8/1/0 | 8/1/0 | 5/3/1 | 6/2/1 | 7/2/0 | 8/1/0 |
| **In Total** | **25/11/18** | **25/13/16** | **27/6/16** | **31/18/5** | **42/9/3** | **44/7/3** | **36/9/9** | **44/9/1** |

| Metrics | $\mathcal{A}$-LSMM-CL ($\mathcal{A}$ = BR-KNN) **against** | | | | $\mathcal{A}$-LSMM-CL ($\mathcal{A}$ = ML-KNN) **against** | | | |
|---|---|---|---|---|---|---|---|---|
| | LIFT | RELIAB | WRAP | HOMI | LIFT | RELIAB | WRAP | HOMI |
| *Hamming Loss* | 3/6/0 | 7/2/0 | 4/4/1 | 6/3/0 | 2/7/0 | 7/2/0 | 3/4/2 | 6/3/0 |
| *Ranking Loss* | 2/0/7 | 2/1/6 | 2/0/7 | 2/0/6 | 8/0/1 | 8/0/1 | 8/0/1 | 8/1/0 |
| *Coverage* | 2/1/6 | 4/1/4 | 4/0/5 | 2/3/4 | 4/3/2 | 7/2/0 | 8/1/0 | 7/1/1 |
| *Average precision* | 3/1/5 | 3/3/3 | 3/1/2 | 4/2/3 | 7/1/1 | 7/2/0 | 5/3/1 | 9/0/0 |
| *Macro-F1* | 8/1/0 | 6/3/0 | 6/1/2 | 6/2/1 | 8/1/0 | 8/1/0 | 7/1/1 | 7/2/0 |
| *Macro-averaging AUC* | 7/2/0 | 7/2/0 | 9/0/0 | 9/0/0 | 8/1/0 | 8/1/0 | 9/0/0 | 9/0/0 |
| **In Total** | **25/11/18** | **29/12/13** | **28/6/15** | **29/10/14** | **36/14/4** | **45/8/1** | **40/9/5** | **46/7/1** |

Table B.4: Win/tie/loss counts (pairwise $t$-test at 0.05 significant level) for $\mathcal{A}$ ($\mathcal{A} \in \{$BR-KNN, ML-KNN$\}$) coupled with LSMM-SE and LSMM-CL against other compared state-of-the-art non-metric learning multi-label classification approaches.